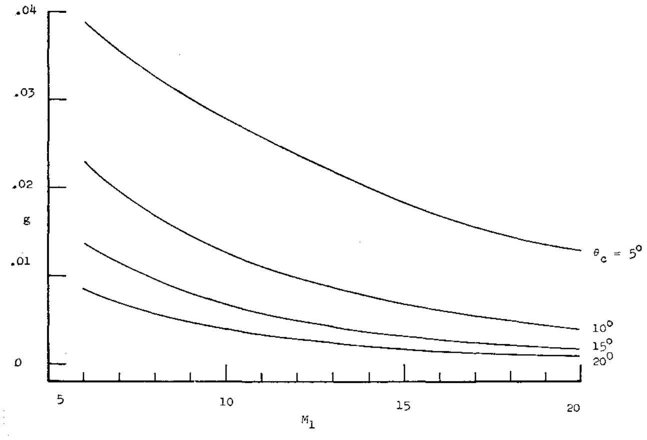

Fig. 2 g vs freestream Mach number M_1.

can now be computed from tables of Kopal, Sims,[6] or from Lees' similarity result.

The numerical problem was solved by an inverse procedure on IBM 7090 by selecting M_1, θ_c, T_w/T_{aw}, θ_δ, and by computing θ_2, $\bar{\chi}_1$ and $\Delta p/p_c$ from Eqs. (1–3). The results were found to be nearly linear with $\bar{\chi}_1$ (for constant M_1, θ_c, and T_w/T_{aw}) and can be expressed as

$$\Delta p/p_c = (fT_w/T_{aw} + g)\bar{\chi}_1$$

where

$$f = f(M_1, \theta_c) \qquad \text{Fig. 1}$$
$$g = g(M_1, \theta_c) \qquad \text{Fig. 2}$$

For $\theta_c > 10°$, linear interpolation will suffice, otherwise a crossplot (e.g., f vs θ_c at given M_1) is needed.

2. Experimental Results

A 10° semivertex angle sharp nosed cone (nose radius 0.001 in.) was tested at zero angle of attack at a nominal Mach number of 14 in the ARL 20-in. wind tunnel. The model was an uncooled tellurium copper cone with the wall thickness 0.125 in. to assure a nearly isothermal surface. The 7-in.-long model contained 7 pressure ports 0.0425 in. in diameter and 2 chromel-alumel thermocouples. Pressure was measured by Hastings-Raydist DV-13 heat conducting gages, and it was recorded on the microsadic magnetic tape, thus permitting individual before-after calibration checks. The tests were conducted at nominal stagnation pressures of 800, 1200, and 1600 psia with stagnation temperatures varying between 1800° and 1900°R. The method of data recording and tunnel conditions are available in detail in Refs. 5 and 7.

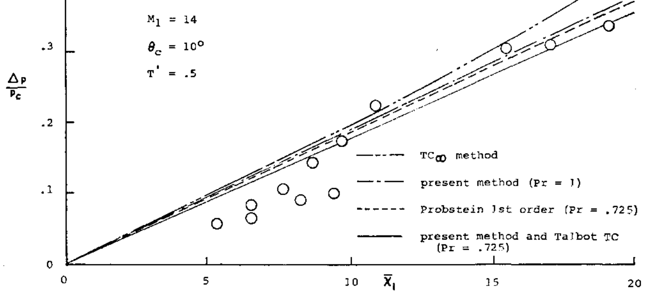

Fig. 3 Induced pressure rise for a 10° cone.

The results are indicated in Fig. 3. The greatest error in the prediction of the induced pressure increment (solid line) corresponds to an error in the surface pressure reading of at most 7%. Considering the magnitude of the static pressures that had to be measured (of order of 1 mm Hg) and the available instrumentation (sensitivity 0.1 mm Hg), it is concluded that all points fall well within the experimental scatter.

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Thermodynamic Properties for Imperfect Air and Nitrogen to 15,000°K

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The recently published thermodynamic properties (T, $\log\rho$, E/RT, H/RT, S/R, $\log p$, Z) for imperfect‡ air and nitrogen as given by Hilsenrath and Klein[1,2] have been used to compute the specific heat (C_p/R, C_v/R, and $\gamma = C_p/C_v$) and speed of sound [$\gamma_E \equiv (\partial \log p/\partial \log\rho)_S$, $a = (\gamma_E\, p/\rho)^{1/2}$] data by Lewis and Neel.[3,4] The data for air§ and nitrogen are given in the ranges $T = 1500(100)15{,}000°K$, $\log\rho = -7(0.2)\,2.2$ and $T = 2000(100)15{,}000°K$, $\log\rho = -7(0.2)2.4$. The data of Hilsenrath and Klein differ from the previously reported perfect-gas data of Hilsenrath, Klein, and Woolley[5] at densities where the effects of intermolecular forces are important. In general, imperfect gas effects are important only at densities above 1 amagat, i.e., the nondimensional density $\rho = \bar{\rho}/\bar{\rho}_a$ where $\bar{\rho}_a$ is the density at 1 atm and $T = 273.15°K$.

The specific heat and speed of sound data for nitrogen in the ranges $T = 100(100)2200°K$ and $\log\rho = -7(0.2)2.4$ have

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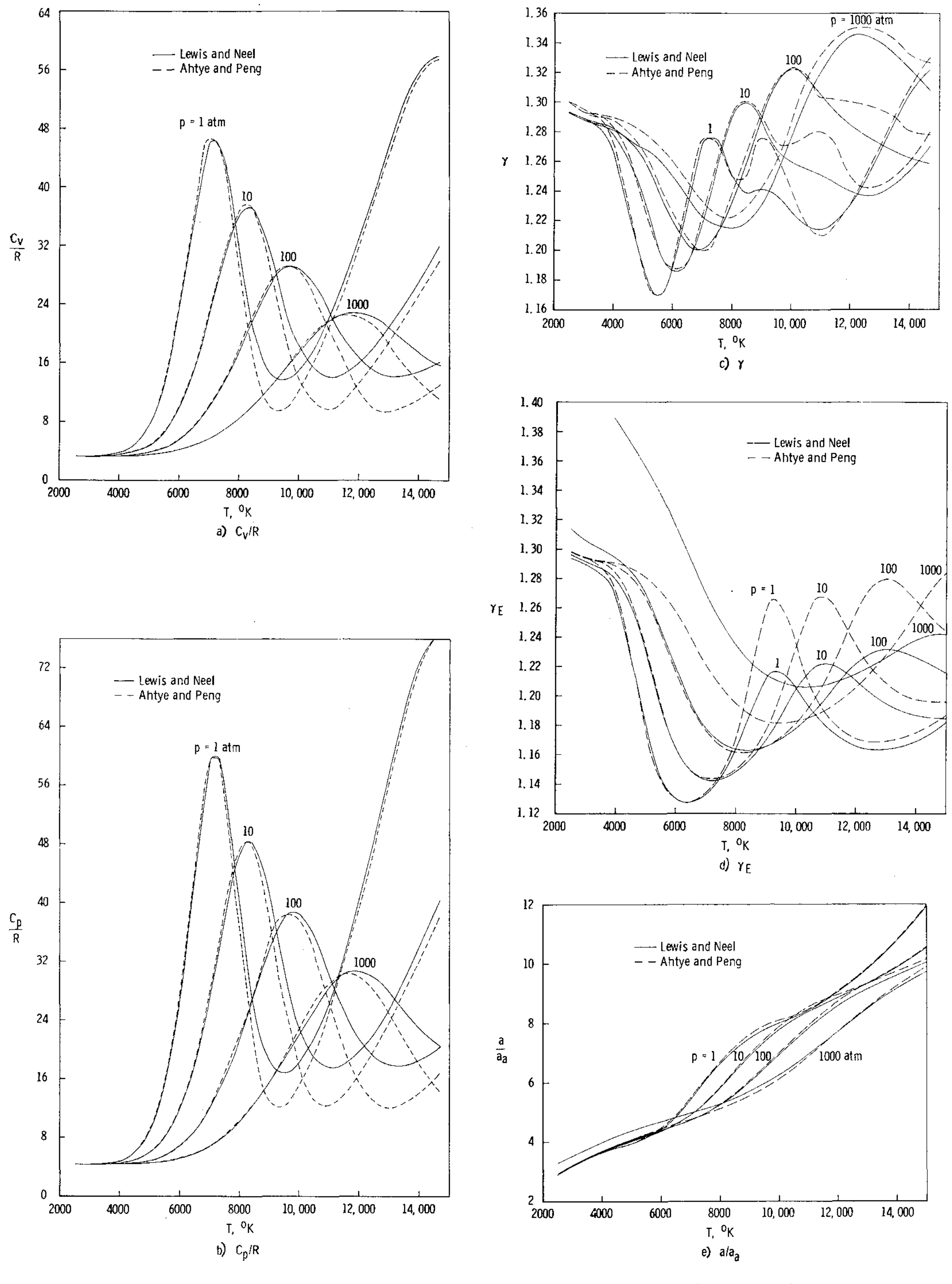
‡ A perfect gas will denote one obeying $p = Z\rho RT$ which includes dissociation and ionization neglecting intermolecular effects. An imperfect gas obeys $p = Z\rho RT$ but includes dissociation, ionization and intermolecular (unless otherwise noted only two-body interaction) forces.

§ A multicolored Mollier diagram for imperfect air has been prepared by members of the staff of the von Karman Gas Dynamics Facility and is available on written request.



a) C_v/R

b) C_p/R

c) γ

d) γ_E

e) a/a_a

Fig. 1 A comparison between the perfect nitrogen results of Ahtye and Peng and the imperfect nitrogen results of Lewis and Neel. Gas constant R = 3196.79 ft²/sec²-°K and the speed of sound a_a = 1105.66 fps.

also been computed by Lewis and Neel[6] based on the data of Little and Neel[7] and Smith[8,9] where the latter data were modified to include only the second virial coefficient $B(T)$ for compatibility with Hilsenrath and Klein[2] at $T = 2000°K$.

The imperfect air specific heat and speed of sound data[3] were compared with the perfect gas results of Landis and Nilson.[10] The comparison showed negligible differences in the specific heat data for the entire density range $\log\rho = -7$ to 2.2 and $T = 1500°$ to 10,000°K. Differences were found at $T > 10{,}000°K$, and they were attributed to differences in the tabulated thermodynamic data. The speed of sound data were affected at $\log\rho > 0$, and the differences increased as ρ increased.

The imperfect nitrogen specific heat and speed of sound data[4] were compared with the perfect gas results of Ahtye and Peng,[11] and those comparisons are shown on Fig. 1. The differences were again found to be modest; however, the Ahtye and Peng data are limited to 1000 atm whereas the Hilsenrath and Klein data extend up to a maximum pressure of 2420 atm, and the imperfect gas effects will increase as the pressure increases above the range shown on Fig. 1.

For applications where speed of sound data are required at high density, the data of Lewis and Neel[3,4] are considered a significant improvement over previously available perfect gas results.

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Eigenvalues for the Equation of Species Conservation with Heterogeneous Reaction

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Fox and Libby[1] consider several problems involving the energy equation for laminar boundary layers with velocity fields described by the Blasius function $f_0(\eta)$; in particular they present solutions, which are in principle exact, to the following two initial value problems. Consider the stagnation enthalpy distribution $g(s, \eta)$, $g \equiv h_s/h_{s,e}$, in a boundary layer wherein the $\rho\mu$ ratio is constant ($C \simeq 1$) and the Prandtl number is unity, and where, at an initial station $s = s_0 > 0$, an arbitrary distribution of stagnation enthalpy is given, i.e., $g(s_0, \eta)$ is specified. The two different problems arise because there are considered two boundary conditions at the wall in the downstream region $s > s_0$. In one, the wall enthalpy in terms of g_w is constant, and in the second, the heat transfer [more precisely $(\partial g/\partial\eta)_w$] is zero. The solution to each of these problems is given in terms of a complete set of orthogonal functions, the first ten of which in each set are presented in Ref. 1.

It is also indicated there that the solutions for the energy distribution just outlined can be applied to certain related problems involving species and/or element conservation. For this application it is necessary to make the additional assumption that a single diffusion coefficient exists with a Schmidt number based thereon equal to unity. The two boundary conditions correspond, respectively, to a fully catalytic surface and to a noncatalytic surface. The question of whether conservation of species or of elements is described depends, of course, on whether or not gas phase reaction exists.

It is the purpose of this note to outline an extension of these considerations to the case of boundary layers with no gas phase reaction but with finite surface catalyticity in the downstream region $s > s_0$ and to present a number of eigenvalues that pertain thereto. As will be discussed below, to obtain the related eigenfunctions involves little numerical effort; thus, the presentation of the eigenvalues, whose determination is not so simple, is considered to be of some value.

For orientation, consider the following mathematical problem:

$$\frac{\partial^3 Y_i}{\partial \eta^3} + f_0 \frac{\partial Y_i}{\partial \eta} - 2sf_0' \frac{\partial Y_i}{\partial s} = 0 \qquad (1)$$

$$Y_i(s_0, \eta) = Y_{i,0}(\eta), \text{ given arbitrary function}$$

$$Y_i(s, \infty) = Y_{i,e}, \text{ given const}$$

$$\frac{\partial Y_i}{\partial \eta}(s, 0) = \zeta_i Y_i(s, 0) \qquad \zeta_i, \text{ given const}$$

where Y_i is the mass fraction of species i, s and η are the usual Levy-Lees variables, and ζ_i the surface catalyticity. Solution of this mathematical problem describes the concentration distribution of species i in a laminar boundary layer which has velocities given by the Blasius function, flows over a surface with a constant surface catalyticity, and which has an arbitrary initial profile of concentration. Such a profile can be created, for example, by an upstream value of surface catalyticity different from ζ_i. Related problems have been under widespread attack; Ref. 2 is especially relevant to the present note, whereas Ref. 3 provides a current review of flows with heterogeneous reactions. Reference 4 provides, inter alia, a review of recent developments in the theory of laminar boundary layers involving gas phase and surface reactions. The reader is referred to this literature for a discussion of the physical aspects of the flow described here.

A solution to the foregoing problem may be obtained by separation of variables. In the interest of brevity the solution will here be simply written down; however, it is easy to confirm that the solution is

$$Y_i = Y_{i,e}(f_{0,w}'' + \zeta_i f_0')(f_{0,w}'' + \zeta_i)^{-1} + \sum_{n=1}^{\infty} A_{n,i} \left(\frac{s}{s_0}\right)^{-\lambda_{n,i}} N_{n,i}(\eta) \qquad (2)$$

where the $A_{n,i}$ coefficients are determined as shown below from the initial conditions, i.e., at $s = s_0$, and where $\lambda_{n,i}$ and $N_{n,i}$ are the eigenvalues and eigenfunctions, respectively, and are defined by the equation

$$N_{n,i}'' + f_0 N_{n,i}' + \lambda_{n,i} f_0' N_{n,i} = 0 \qquad (3)$$

subject to the conditions

$$N'_{n,i}(0) = \zeta_i N_{n,i}(0) \qquad N_{n,i}(\infty) = 0 \qquad (4)$$

The functions $N_{n,i}(\eta)$ defined by Eqs. (3) and (4) are generalizations of those previously presented in Ref. 1, since for $\zeta_i \to \infty$ there result the eigenfunctions for the case of constant wall enthalpy and for $\zeta_i = 0$ the eigenfunctions for the case of an adiabatic surface.

As demonstrated in Ref. 1, well-known procedures can be used to show that, if exponential decay to zero of the eigenfunctions is required as $\eta \to \infty$, then these functions form a complete orthogonal set and the eigenvalues are real positive numbers. Thus, as $s/s_0 \to \infty$ in Eq. (2), the solution becomes that corresponding to a Crocco relation between concentration and velocity. Moreover, the orthogonality condition is found to be

$$\int_0^\infty \left(\frac{f_0'}{f_0''}\right) N_{n,i} N_{m,i} \, d\eta = C_{n,i} \delta_{mn} \qquad (5)$$

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